and quartz windows. The quartz window can withstand relative pressures up to 2 kg/cm^2 (we experienced quartz failure once at 3.5 kg/cm^2 and once at 4 kg/cm^2). The teflon window has already withstood a relative pressure of 4.5 kg/cm^2, but further tests are still in progress.

Conclusion

The principles of operation of a traveling-wave resonator designed by the authors have been described. Constituent elements have been discussed and the application to high-power tests was mentioned. Now the fact is stressed that power-handling testing is not the sole application of traveling-wave resonators. This circuit may also be used for other types of measurements. For instance, in the case of optimum coupling, attenuation introduced in the main waveguide is infinite and variation of attenuation within main waveguide aorund this value is very rapid. This can be applied to measurement of low variations in attenuation when studying surface treatments for waveguide, gas tube losses, etc.

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Reflection Coefficient of *E*-Plane Tapered Waveguides*

KATSU MATSUMARU†

Summary—This paper treats the reflection of linearly and sinusoidally tapered waveguides. In the first part, reflection coefficients of linearly tapered waveguides for dominant modes are calculated. Graphs of the vswr of tapers for different impedance ratios are plotted showing that the vswr does not go to unity at multiples of a half wavelength. In the second part, reflection coefficients of sinusoidally tapered waveguides are calculated. Experimental data verify the theory for both kinds of tapers of various lengths at 4 kmc band.

Linear tapers perform almost as well as exponential tapers, and better than shorter hyperbolic tapers. The reflection coefficients of sinusoidal tapers can be about half as small as that of the linear tapers, and these tapers compare favorably with the Dolph-Tchebycheff and the Willis taper of improved design.

Introduction

Reflection coefficients of tapered waveguides can be calculated by formulas described in the references,[1,2] but these formulas give only rough values. Reflections of several nonuniform transmission lines were theoretically treated by Burrow,[3] Scott,[4] Klopfenstein,[5] Collin,[6,7] Willis and Sinha,[8] Bolinder,[9,10] and others. Most papers treat the magnitudes of reflection coefficient of particular tapers that are mathematically convenient to analyze, but in microwave circuit we often need practical formulas and convenient graphs to determine the reflection coefficients. These several papers are mainly theoretical, with very limited experimental data on tapered waveguides.

In the first part of this paper, approximate theoretical calculations of the reflection coefficient of linear tapers are presented. From the derived formulas, useful graphs were compiled in terms of the suitable ratios of input-to-output surge impedances. To confirm the formulas experimentally at 4 kmc, we have made two groups of tapers in which the ratios of surge impedances are 2.0 and 2.4. The agreement between calculated and

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1 T. Moreno, "Microwave Transmission Design Data," McGraw-Hill Book Co., Inc., New York, N. Y., p. 53; 1948.

2 G. L. Ragan, "Microwave Transmission Circuit," McGraw-Hill Book Co., Inc., New York, N. Y., M.I.T. Rad. Lab. Ser., vol. 9, p. 305; 1948.

3 C. R. Burrow, "The exponential transmission line," *Bell Sys. Tech. J.*, vol. 17, pp. 555–573; October, 1938.

4 H. J. Scott, "The hyperbolic transmission line as a matching section," Proc. IRE, vol. 41, pp. 1654–1657; November, 1953.

5 R. W. Klopfenstein, "A transmission line taper of improved design," Proc. IRE, vol. 44, pp. 31–35; January, 1956.

6 R. E. Collin, "The theory and design of wide-band multisection quarter-wave transformer," Proc. IRE, vol. 43, pp. 179–185; February, 1955.

7 R. E. Collin, "The optimum tapered transmission line matching section," Proc. IRE, vol. 44, pp. 539–548; April, 1956.

8 J. Willis and N. K. Sinha, "Non-uniform transmission lines as impedance transformers," *Proc. IEE*, pt. B, vol. 103, pp. 166–172; March, 1956.

9 F. Bolinder, "Fourier transformers in the theory of inhomogeneous transmission lines," Proc. IRE, vol. 38, p. 1354; November, 1950.

10 F. Bolinder, "Fourier transforms in the theory of inhomogeneous transmission lines," *Trans. Roy. Inst. Tech., Stockholm*, No. 48; 1951.

measured values of reflection coefficient was found to be very good.

A sinusoidally tapered waveguide may be used to shorten the length of the taper. In the later part of this paper general formulas for the reflection coefficients of these tapers are described. To confirm the formulas, we made a third group of tapers in which the ratio of input-to-output surge impedance is 2.4. The calculated curve of reflection coefficient of these tapers agreed very well with that of the observed. Bolinder was the first to derive accurate formulas for linear and sinusoidal tapers. While his results are in terms of magnitude of the reflection coefficient, we have derived explicit formulas of *complex* reflection coefficients of tapered waveguides. By means of our practical charts, we can easily determine the exact reflection coefficient of a tapered waveguide.

Calculations and Characteristics of Reflection Coefficient of Linearly Tapered Waveguide

In Fig. 1(a), W_1 and W_2 are rectangular waveguides, which are connected by an E-plane linear taper. The surge impedances of W_1 and W_2 and the taper are Z_1, Z_2, and $Z(x)$, respectively. The origin of x is at the longitudinal center of the taper, and the length of the taper is l. Let us assume that the $Z(x)$ is a linear function of the variable x and Z_1 is larger than Z_2, *viz.*,

$$Z_1 > Z_2, \tag{1}$$

$$Z(x) = \frac{Z_1 + Z_2}{2} + \frac{Z_2 - Z_1}{l} x. \tag{2}$$

Furthermore, we assume that Z_1, Z_2, and $Z(x)$ are positive real. In Fig. 1(b) two transmission circuits whose surge impedances are Z_1' and Z_2', are connected by a step discontinuity. If the electromagnetic wave propagates from left to right, the element of reflection coefficient dR is given by (3) when Z_1' is only slightly different from Z_2'. It is

$$dR = \frac{Z_2' - Z_1'}{Z_2' + Z_1'} = \frac{dZ_1'}{2Z_1'}. \tag{3}$$

By considering the phase difference, we can obtain R of the taper of Fig. 1(a) by the following integration procedure. With

$$dZ = \frac{Z_2 - Z_1}{l} dx,$$

$$R = \frac{1}{2}\int_{-l/2}^{l/2} \frac{e^{-j2\beta(x+l/2)}}{\frac{Z_1 + Z_2}{2} + \frac{Z_2 - Z_1}{l} x} \cdot \frac{Z_2 - Z_1}{l} dx. \tag{4}$$

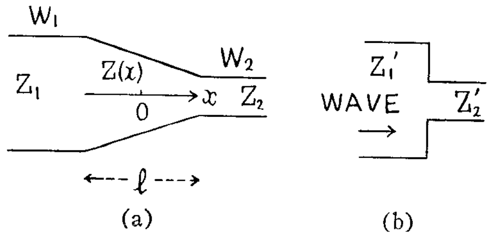

Fig. 1—Illustration of the coordinate in a taper.

The wavelength in the taper, λ_g is constant, and β is $2\pi/\lambda_g$. Putting

$$q \equiv \frac{2(Z_2 - Z_1)}{(Z_1 + Z_2)l},$$

(4) becomes

$$R = \frac{Z_2 - Z_1}{(Z_2 + Z_1)l}\int_{-l/2}^{l/2} \frac{e^{-j2\beta(x+l/2)}}{1 + qx} dx. \tag{5}$$

As a first approximation, we consider (5) when q is negligibly small. In this case, the taper may be considered an exponential one. Then,

$$|R| = \frac{1}{2}\ln\frac{Z_1}{Z_2} \cdot \frac{|\sin(2\pi l/\lambda_g)|}{2\pi l/\lambda_g}. \tag{6}$$

This result is identical with the equation given by Ragan.[2] Values of $|R|$ derived from (6) show that the reflection coefficient reduces to zero at the minimum points. Eq. (6) is not applicable for general cases when q is not equal to zero. To acquire a more accurate and useful equation, we must integrate (5) by considering q properly.

By using the approximate relation,

$$\frac{1}{1 + qx} \doteqdot 1 - qx + q^2x^2, \tag{7}$$

we have from (5)

$$R = \frac{Z_2 - Z_1}{(Z_2 + Z_1)\beta l}\left\{\begin{array}{l} \sin\beta l \cdot \cos\beta l(1 + q^2l^2/4) \\ \quad + (l\cos\beta l - \sin\beta l/\beta) \\ \quad \times(-q\sin\beta l/2 + q^2\cos\beta l/2\beta) \\ \quad -j\left\{\begin{array}{l} \sin^2\beta l(1 + q^2l^2/4) \\ + (l\cos\beta l - \sin\beta l/\beta) \\ \times(q\cos\beta l/2 + q^2\sin\beta l/2\beta) \end{array}\right\} \end{array}\right\}. \tag{8}$$

This equation is identical with (6) when we put $q = 0$. From the explicit formula (8) we can easily calculate a magnitude of R of a taper. Eq. (8) was derived on the base of assumption (1). We may also use this formula for the case of $Z_2 > Z_1$. However, except for extremely short tapers, the calculated magnitudes of R are equal for both the input and output sides.

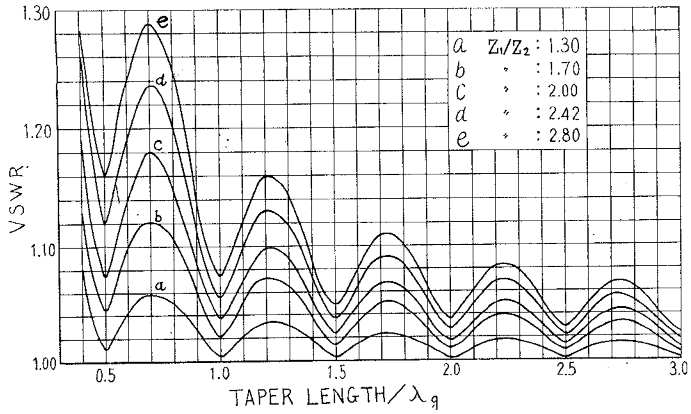

Fig. 2—Curves show vswr's of linearly tapered waveguides calculated from (8) with Z_1/Z_2 as the parameter.

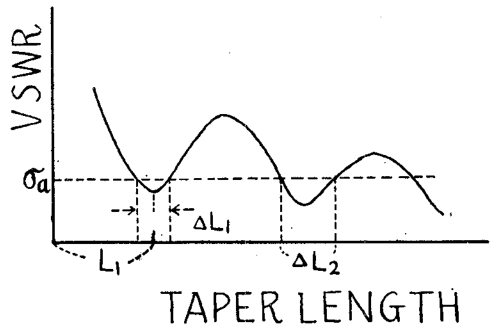

Fig. 3—Method of taper design.

Placing values of Z_1, Z_2, and l in (8), we can calculate the vswr of a taper from the relationship,

$$\text{vswr} = \frac{1 + |R|}{1 - |R|}.$$

Fig. 2 shows graphs of calculated vswr's for various values of Z_1/Z_2. It is to be noted that usually the required bandwidths for microwave systems are relatively narrow, thus the method of design of a taper will be mentioned here. Let us suppose that for a given value of Z_1/Z_2, the vswr is as shown in Fig. 3. If the allowable value of the vswr is σ_a, we draw a horizontal line across σ_a. By estimating widths of ΔL_1, ΔL_2, etc., we can determine the necessary length of the taper.

Now, we compare the reflection characteristics of a linear taper with that of others. First, we compare the $|R|$ calculated from (8) with that given in Bolinder's paper.[11] In Table I, a very good comparison is shown for the case of $Z_1/Z_2 = 2.0$. Bolinder's formula is expressed in terms of complicated Bessel functions and gives only the magnitude of R. It is believed that our formula foı R in (8), being simpler and virtually as accurate, will prove to be more practical for designing of a taper. Next, Klopfenstein[5] described the performances of exponential, hyperbolic, and Dolph-Tchebycheff coaxial tapers for the case of $Z_1/Z_2 = 1.5$. Fig. 4 compares the performance of the linear taper with these

TABLE I

$l/\lambda g$		0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
$\|R\|$	(8)	0.28	0.18	0.09	0.04	0.07	0.08	0.07	0.04	0.02	0.03	0.05	0.04	0.03
	Bolinder	0.27	0.17	0.09	0.04	0.07	0.08	0.07	0.04	0.03	0.04	0.05	0.04	0.03

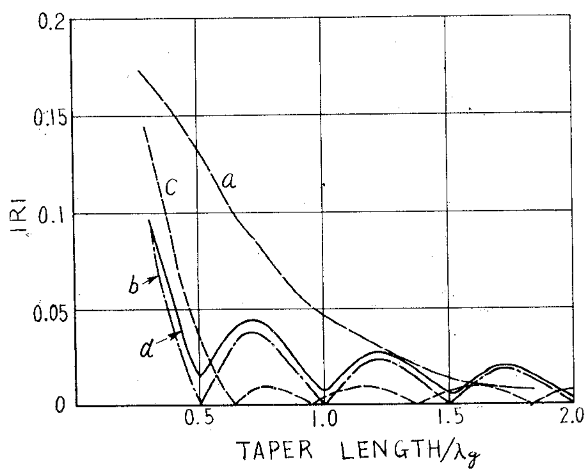

Fig. 4—Performances of linear taper ($Z_1/Z_2 = 1.5$). a:hyperbolic, b:exponential, c:Dolph-Tchebycheff, d:linear.

three other types. It is shown that the reflection of a linear taper is not too different from an exponential one. The reflection of the Dolph-Tchebycheff taper is shown to be much better than the linear, but this should be confirmed experimentally. The hyperbolic taper is disadvantageous for waveguide applicatons. Briefly the conclusion is that, in most cases, a linear taper may be substituted for an exponential.

For microwaves applications, one should use a reflection formula of a tapered transmission line which has been experimentally verified. The reasons are as follows. Most theoretical formulas are approximations derived on the bases of assumptions. For example, the waves propagating in waveguides are not TEM in character. The definition of a surge impedance of a wave-

[11] Bolinder, *ibid.*, p. 67 (Fig. 27).

guide is merely formal. Strictly speaking, the equivalent circuit of a taper is composed of many reactances, and hence it is frequency dependent. Moreover, higher modes may be induced in the output or input terminals. For example, Lewin[12] treated the reflection cancellation at junctions between waveguides and horns. The calculated values of reflection coefficient cannot be accurate for extremely short tapers, of course, and it is important to determine the minimum lengths for which the formulas may be used. For these reasons, we determined to get experimental data. We performed experiments at 4 kmc, because this band is most convenient in view of the precision of measurements available at our laboratory.

Experiments—Part I ($Z_2/Z_1=2.0$)

For a dominant mode in a waveguide of constant width a, the surge impedance is proportional to the height b.[13] The first experiments were made for the case that Z_2/Z_1 is 2.0. That is, the internal sizes, b and a, of the waveguides were 58.1 mm $\times$58.1 mm and 29.1 mm $\times$58.1 mm. Lengths of tapers were from 17 cm to 2 cm at intervals of 1 cm. Accuracy of internal sizes in the plane of flanges was approximately ± 0.2 mm. The vswr was measured on the rectangular side using a specially made sliding wooden dummy load on the square side. The residual vswr of this dummy load was approximately 1.01 at this band. We eliminated errors due to this load by shifting its position. The law of the crystal on the standing-wave device was measured. The data were taken at a frequency of 4 kmc ($\lambda_g=9.82$ cm). Observed and calculated results are shown in Fig. 5, together with other data. Observed points shown by white circles coincide well with the dashed curve for tapers longer than 0.5 λ_g. The mean error in the vswr of the tapers longer than 0.5 λ_g was as small as 0.005. In this example Z_2 is larger than Z_1, so that, the calculated magnitudes of vswr are not equal to the curve c shown in Fig. 2. However, the deviations of vswr from that curve are extremely small. We may use Fig. 2 for this example. It is believed that graphs of Fig. 2 are generally inapplicable for tapers shorter than 0.5 λg. So, it is important to determine the minimum lengths for which other authors' formulas may be used.

Experiments—Part II ($Z_1/Z_2=2.4$)

We measured vswr's of another group of tapers connecting waveguides with internal dimensions of 29.1 mm $\times$58.1 mm and 12.0 mm $\times$58.1 mm, for which Z_1/Z_2 is almost 2.4. Lengths of this group are from 20 cm to 3 cm at intervals of 1 cm. The constructional accuracy of this group is not as good as the first group; namely almost ± 0.35 mm. Measurements were made at a frequency of 3960 mc; that is, λ_g was almost 10.0 cm. Fig. 5 shows the results. Black circles show observed values and the solid curve the calculated. In this group, again, the results coincide well for tapers longer than $0.5\lambda_g$. Deviations are slightly greater than the first group; the mean error of the vswr is as small as 0.008.

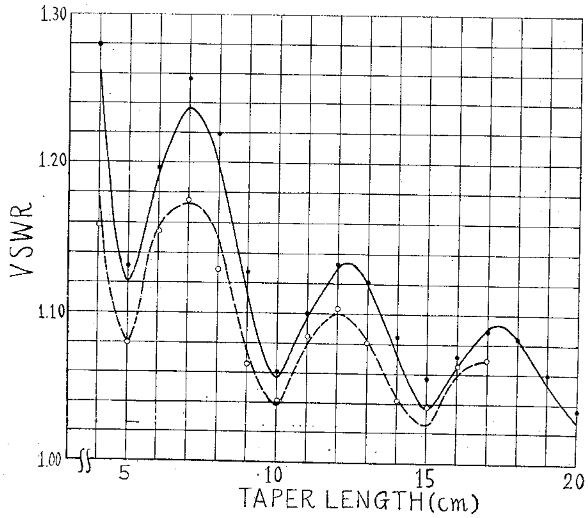

Fig. 5—Results of measurements of vswr of linear tapers. White circles show observed values, and the dashed curve the calculated. The ratio of Z_2/Z_1 is 2.0. Black circles and the solid curve show the corresponding values for the ratio of Z_1/Z_2 equal to 2.4.

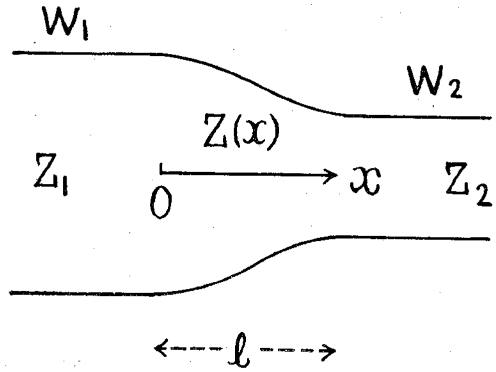

Fig. 6—Illustration of the coordinate in a taper.

Calculation of Reflection Coefficient of Sinusoidally Tapered Waveguide

We calculate the reflection coefficient of a sinusoidally tapered waveguide for the dominant mode in the same way as that for the linearly tapered waveguide. The width a of a rectangular waveguide is constant, and its height b varies sinusoidally as shown in Fig. 6. We assume that Z_1, Z_2, and $Z(x)$ are positive real and Z_1 is greater than Z_2, thus,

$$Z_1 > Z_2. \tag{9}$$

$$Z(x) = \frac{Z_1 + Z_2}{2} - \frac{Z_2 - Z_1}{2} \cos\left(\frac{\pi x}{l}\right). \tag{10}$$

From (10),

[12] L. Lewin, "Reflection cancellation in waveguides," *Wireless Eng.*, vol. 10, pp. 258–264; August, 1949.

[13] A. Schelkunoff, "Electromagnetic Waves," D. Van Nostrand Co., Inc., New York, N. Y., pp. 316–322; 1943.

$$dZ(x) = \frac{\pi(Z_2 - Z_1)}{2l} \sin\left(\frac{\pi x}{l}\right) dx. \tag{11}$$

Thus, if the electromagnetic wave propagates from left to right, R of the taper in Fig. 6 can be obtained, after considering the phase factor, from

$$R = \frac{\pi(Z_2 - Z_1)}{2l(Z_2 + Z_1)} \int_0^l \frac{e^{-j2\beta x} \sin(\pi x/l)}{1 - \frac{Z_2 - Z_1}{Z_2 + Z_1}\cos\left(\frac{\pi x}{l}\right)} dx. \tag{12}$$

To evaluate this integral, we put

$$r \equiv \frac{Z_1 - Z_2}{Z_2 + Z_1}.$$

As $|r|$ is smaller than unity, we have

$$\frac{1}{1 + r\cos\left(\frac{\pi x}{l}\right)} \cong 1 - r\cos\left(\frac{\pi x}{l}\right) + r^2\cos^2\left(\frac{\pi x}{l}\right). \tag{13}$$

Using this approximate relation, we obtain

$$R = \frac{\pi(Z_2 - Z_1)}{2l(Z_2 + Z_1)} \int_0^l e^{-j2\beta x} \sin\left(\frac{\pi x}{l}\right) \cdot \left\{1 - r\cos\left(\frac{\pi x}{l}\right) + r^2\cos^2\left(\frac{\pi x}{l}\right)\right\} dx. \tag{14}$$

On evaluating (14) we find

It is clear that (15) is a function of r and l/λ_g. As a check, we put $r=0$ in (14), and (15) is given by

$$R = \frac{1}{2}\frac{Z_2 - Z_1}{Z_2 + Z_1}\cos\left(\frac{2\pi l}{\lambda_g}\right) \times \left\{\cos\left(\frac{2\pi l}{\lambda_g}\right) - j\sin\left(\frac{2\pi l}{\lambda_g}\right)\right\} \times \left\{\frac{\pi}{l} \Big/ \left\{\frac{4\pi}{\lambda_g} + \frac{\pi}{l}\right\} - \frac{\pi}{l} \Big/ \left\{\frac{4\pi}{\lambda_g} - \frac{\pi}{l}\right\}\right\}.$$

In this equation, if l tends to zero, R becomes

$$R = \frac{Z_2 - Z_1}{Z_2 + Z_1}.$$

So, (15) converges to the reflection coefficient of two directly connected waveguides. In Fig. 7, curves show vswr's for various values of Z_1/Z_2, corresponding to those in Fig. 2. It should be mentioned that the curves of the vswr do not go to unity at the minimum points and their maximum points ascend in accordance with the ratios of Z_1/Z_2. As for a linear taper, Fig. 7 may be used for the case of $Z_2 > Z_1$. The reflection character described in this chart shows clearly the very high quality of these tapers. It is mentioned that the vswr's increase suddenly for tapers shorter than $0.7\lambda g$.

$$R = \frac{\pi(Z_2 - Z_1)}{2l(Z_2 + Z_1)} \left[\begin{array}{l} \cos^2\left(\frac{2\pi l}{\lambda_g}\right)\left\{\left(1 + \frac{r^2}{4}\right)\frac{-\frac{2\pi}{l}}{\left(\frac{4\pi}{\lambda_g}\right)^2 - \left(\frac{\pi}{l}\right)^2} + \frac{r^2}{4}\cdot\frac{-\frac{6\pi}{l}}{\left(\frac{4\pi}{\lambda_g}\right)^2 - \left(\frac{3\pi}{l}\right)^2}\right\} \\ + \frac{r}{2}\sin^2\left(\frac{2\pi l}{\lambda_g}\right)\frac{\frac{4\pi}{l}}{\left(\frac{4\pi}{\lambda_g}\right)^2 - \left(\frac{2\pi}{l}\right)^2} \\ + j\sin\left(\frac{2\pi l}{\lambda_g}\right)\cos\left(\frac{2\pi l}{\lambda_g}\right)\left\{\left(1 + \frac{r^2}{4}\right)\frac{\frac{2\pi}{l}}{\left(\frac{4\pi}{\lambda_g}\right)^2 - \left(\frac{\pi}{l}\right)^2} + \frac{r^2}{4}\frac{\frac{6\pi}{l}}{\left(\frac{4\pi}{\lambda_g}\right)^2 - \left(\frac{3\pi}{l}\right)^2} + \frac{r}{2}\frac{\frac{4\pi}{l}}{\left(\frac{4\pi}{\lambda_g}\right)^2 - \left(\frac{2\pi}{l_y}\right)^2}\right\} \end{array} \right]. \tag{15}$$

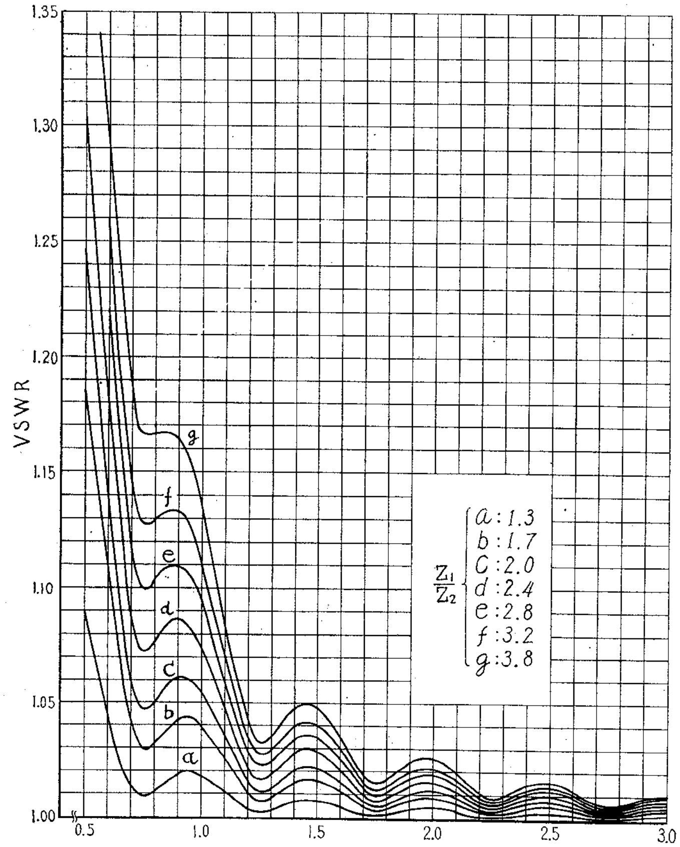

Fig. 7—Curves show vswr's of sinusoidally tapered waveguide as the parameter of Z_1/Z_2. The impedance ratios are 1.3 to 3.8.

Thus, for approximately equivalent performance, the sinusoidal taper can be made about half as long as the linear taper. Next, we compare the sinusoidal taper with the taper which was described in Willis and Sinha's paper.[15] In this case, Z_1/Z_2 is equal to 2.0. Fig. 8 shows reflection coefficients of both tapers. The length of the sinusoidal tapers is almost 0.65 λ_g for $|R|$ equal to 0.042 as compared with 0.5 λ_g for the Willis taper, but this is compensated by a faster reflection drop-off in the sinusoidal taper. Lastly, we compare the sinusoidal taper with the taper which was described in Klopfenstein's paper.[16] Fig. 9 shows reflection performances of both tapers. The Dolph-Tchebycheff taper appears to be somewhat better, but not by a major degree. Thus from Figs. 8 and 9 we see that as to the reflection character, a sinusoidal taper compares well with several kinds of tapers designed by more difficult methods. Consequently, for many applications the sinusoidal taper would be preferable from the standpoint of design simplicity coupled with adequate performance.

Experiments—Part III ($Z_1/Z_2=2.4$)

As the second group, we experimented for the case that Z_1/Z_2 is 2.4. To confirm the formulas, it appears most favorable to use taper lengths near the first minimum point of the vswr curve, so that lengths from 0.5 λ_g to 1.0 λ_g with intervals of 1 cm were used. The accuracy of measurements was, on the whole, improved compared to that of the two previous groups. Measurements were also made at 3.96 kmc. The results are expressed in Table IV and show excellent agreement. As shown by the curve d in Fig. 7, the observed values of vswr yield a minimum near the taper length of 0.75 λ_g and a maximum near 0.9 λ_g. It is believed that the mean error in the vswr of tapers longer than 0.8 λ_g is about as small as 0.01.

TABLE II

$l/\lambda g$		0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
VSWR	(15)	1.24	1.14	1.06	1.05	1.06	1.06	1.04	1.02	1.01	1.02	1.02
	Bolinder	1.25	1.13	1.07	1.05	1.06	1.06	1.03	1.01	1.02	1.03	1.02

TABLE III

Length (cm)		10	11	12	13	14	15	16	17	18	19	20
VSWR	Linear.	1.04	1.07	1.10	1.09	1.05	1.02	1.05	1.07	1.06	1.04	1.02
	Sinu.	1.06	1.04	1.02	1.01	1.02	1.02	1.02	1.01	1.01	1.01	1.01

Comparison with Other Tapers

Now, for the case of $Z_1/Z_2=2.0$, we show the vswr's from (15) with those calculated from Bolinder's paper[14] in Table III. The agreement of the two sets of values is very good. We next compare sinusoidal tapers with linear tapers. As an example, let us consider a waveguide wavelength λ_g of 10 cm. The vswr of these two tapers are given in Table III for Z_1/Z_2 equal to 2.0.

[14] Bolinder, *op. cit., Trans. Roy. Inst. Tech., Stockholm,* p. 65 (Fig. 26).

[15] Willis and Sinha, *op. cit.* (Fig. 3).

[16] Klopfenstein, *op. cit.* (Fig. 6).

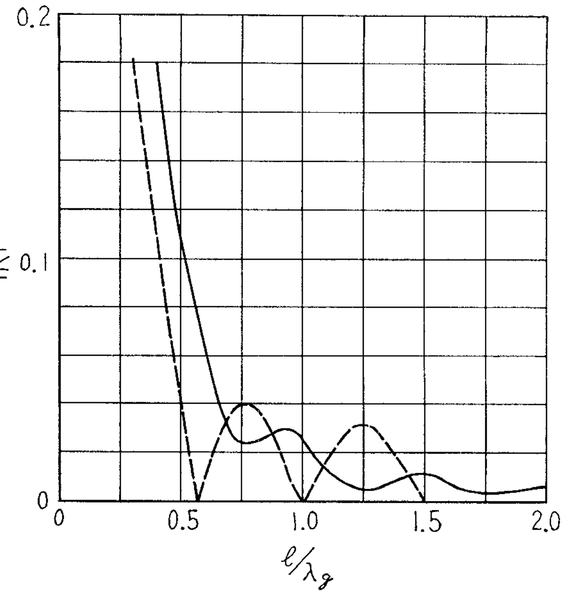

Fig. 8—Reflection coefficient as a function of l/λ_g for the example of Willis and Sinha's paper and sinusoidal taper. Solid line: sinusoidal, and dashed line: the Willis taper.

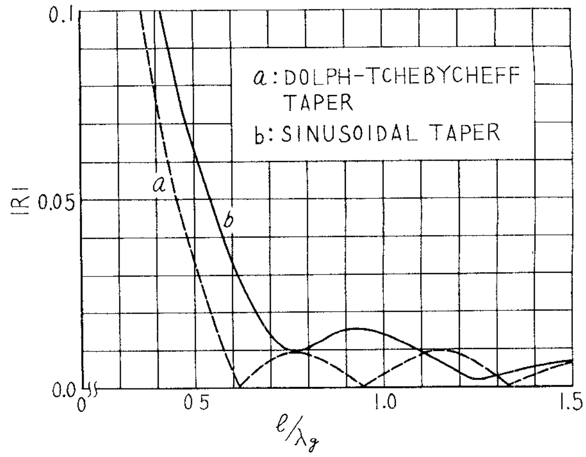

Fig. 9—Reflection coefficient as a function of l/λ_g for the Dolph-Tchebycheff taper and sinusoidal taper.

TABLE IV

$l/\lambda g$		0.5	0.6	0.7	0.8	0.9	1.0
VSWR	Obs.	1.38	1.11	1.06	1.07	1.08	1.07
	Cal.	1.26	1.16	1.08	1.07	1.08	1.07

Conclusion

The reflection coefficient of a linearly and a sinusoidally tapered waveguide can be calculated quite accurately by (8) and (15), respectively. Except for extremely short tapers, the calculated magnitudes of R are equal for both the input and output sides. For most cases, however, we may directly determine vswr's by means of the graphs described in Fig. 2 and 7. Both the observed values of the vswr of linear tapers longer than 0.5 λ_g and sinusoidal tapers longer than 0.8 λ_g coincide with calculated values at most within the limit of 0.01. The effect of neglecting higher terms in (8) and (15) is very small, so that (8) and (15) are generally sufficient for practical uses.

The reflection performance of a linear taper is almost as good as an exponential one, and a sinusoidal taper is comparable to the Dolph-Tchebycheff and the Willis taper of improved design.

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